paleocurrents and intraformational unconformities in the rocks suggest that the integrated Chinle-Dockum depositional system was disrupted when rift-related structures began to form (29).

REFERENCES AND NOTES

2. For example, L. Abbott and T. E. Smith, Geology 17, 329 (1989).
8. Mean $Q_{\text{Fe}}^{\text{tot}}/n = 78 (10)$; J. Schnabl, thesis, Texas Tech University (1994).
15. Approximately 30 kg of the Santa Rosa Sandstone were collected from a cut bank along Sierreta de la Cruz Creek in Potter County, Texas [locality SD01 of (8)]. The sandstone here unconformably overlies the Quadrangle Ranch Formation, a succession of Pennsylvanian red beds. The sample was crushed, and zircons were separated by standard methods. The zircon separate was sieved to >100 μm, and the largest zircons, generally 125 to 150 μm in longest dimension, were picked into six fractions on the basis of color, shape, and lack of inclinations. Grains were abraded for ~4 hours and analyzed individually. Dissolution in 0.1 M HF-microwave capsule within a 125-ml digestion chamber was followed by recycling with a mixed $\text{HF} + \text{UO}_2$ spike and standard chemical separation of $\text{U}$ and $\text{Th}$. The elements were loaded on silica gel onto glass filters and analyzed with a VG-354 mass spectrometer in static mode with the use of three Faraday collectors and a Dally detector system for Pb. The two Faraday collectors for $\text{UO}_2$. The time scale of Harland et al. (13) is used to correlate isotopic and faunal ages.

25. n = 3 in Oslo, compared with $n = 1$ in Santa Rosa Sandstone.
29. T. Lehman, unpublished data.
32. Acknowledgment by N.R.F. is made to the Donors of The Petroleum Research Fund, administered by the American Chemical Society, for support of this research. Support for zircon geochemistry and analysis was provided by National Science Foundation grant EAR-9416903 (O.E.G.). We thank R. F. Dubiel, J. D. Gleason, T. F. Lawton, S. G. Lucas, F. J. Pazzaglia, and R. Slingerland for discussions; T. F. Lawton for review of an earlier version of the manuscript; and J. Ruiz for information about Permian plutons in Mexico. We thank J. P. Schnabl for sample collection and A. L. Roach for zircon separation.

Late Proterozoic and Paleozoic Tides, Retreat of the Moon, and Rotation of the Earth

C. P. Sonett,* E. P. Kvale, A. Zakharian, Marjorie A. Chan, T. M. Demko

The tidal rhythms in the Proterozoic Big Cottonwood Formation (Utah, United States), the Neoproterozoic Elatina Formation of the Flinders Range (southern Australia), and the Lower Pennsylvanian Pottsville Formation (Alabama, United States) and Mansfield Formation (Indiana, United States) indicate that the rate of retreat of the lunar orbit is $df/dt \sim k_2 \sin(\delta)$ (where $\delta$ is the Earth-moon radius vector, $k_2$ is the tidal Love number, and $\delta$ is the tidal lag angle) and that this rate has been approximately constant since the late Precambrian. When the contribution to tidal friction from the sun is taken into account, these data imply that the length of the terrestrial day 900 million years ago was ~18 hours.

The well-known tides induced by Earth on the sun and moon have had several long-term effects over the age of Earth. Most notably, the transfer of angular momentum from Earth to the moon has resulted in an appreciable increase in the length of the day and a retreat of the moon from Earth. Here, we used laminated tidal sediments to determine tidal periods back to 900 million years ago. From these records, the retreat rate of the moon—that is, the evolution in time of the lunar semimajor axis—can be calculated. In principle, the information derived from tidal rhythmologies (tidalites) can also yield the rotational deceleration of Earth, the change in the length of day (LOD), the rate of generation of terrestrial tidal frictional heat, and the variation with time of the product of $k_2$ and $\sin(\delta)$. Tidalites consist of stacked sets (commonly of millimeter to centimeter scale) of laminated mudstone or intercalated beds of sandstone and mudstone; successive sets exhibit progressive vertical thinning and thinning in response to daily changes in current velocities associated with tidal processes. Tidalites from a variety of modern settings—including delta fronts, abandoned tidal channels, tidal flats, and estuaries—have been described (1).

The most common reported tidal cyclicity in the rock record include daily, semidiurnal, and semimonthly periods. Semimonthly (near-spring) periods reflect phase changes of the moon during the half-synodic month and lunar declinational changes associated with the half-tropical month (2). During the synodic month, tides are higher when Earth, the moon, and the sun are nearly aligned (syzygy) and are lower when the radius vectors from Earth to the sun and moon enclose a right angle (quadrature). Spring tides form during syzygy (full and new moon), whereas neap tides form during quadrature (the waxing and waning phases of the moon) (3). Deviations from tidal equilibrium are always encountered in the tidal record (4); these deviations result from local tidal geometry and variable basinal
harmonic response. Ideally, the lunar synodic period is estimated by determining the neap-spring events in a solar year; this approach avoids counting errors arising from possible losses of individual laminae. Effects of ancient basinal harmonics are often difficult to subtract from the rock record. Such corrupting effects can be avoided by restricting analysis to the neap-spring cycle or by using very long records of the individual semidiurnal events. We primarily examined laminae group widths corresponding to neap-spring cycles associated with the half-synodic month.

The tidal sequences we analyzed are preserved in the Big Cottonwood Formation (BCC) in Utah, 900 ± 100 Ma (million years ago) (Fig. 1) (5); the Elatina Formation of southern Australia, 650 ± 100 Ma (6); the Potts ville Formation of Northern Alabama, 312 ± 5 Ma (7); and the Hindostan whetstone beds in the Mansfield Formation of Indiana, 305 ± 5 Ma (8). These records are separated in time by intervals of ~300 million years (My). The Mansfield and Potts ville tidalite ages are based on biostratigraphic and lithostratigraphic correlations; the ages of the Elatina Formation and BCC are estimated from adjoining igneous rocks. The thickness of the tidalites in each formation is variable but is generally a few millimeters. After the cores were halved and polished to optimize contrast, we counted the tidalites with the use of a binocular microscope fitted with a micrometer.

Neap-spring counting previously reported for the Elatina Formation has yielded a mean rate of lunar retreat from the late Precambrian to the present (6). The lunar orbital period has also been inferred from studies of intertidal Devonian corals and other biota (9). These studies suggested that the length of the year was ~400 days at 345 to 395 Ma; this estimate implies a geologically late close approach of the moon to Earth, but there is no evidence of the "megatides" and extraordinary heating that would have resulted (10, 11).

Both the moon and sun induce gravitational quadrupole moments in Earth. The tidal bulge "leaks" the Earth-moon and Earth-sun radius vectors by tidal lag angles. Tides raised by the moon are some five times those raised by the sun; the two tidal perturbations approximately converge at synzygy. Angular momentum from Earth's spin couples to both the lunar orbit and (to a lesser extent) Earth's orbit about the sun. The transfer of angular momentum to the moon's orbit leads to an increase of the major axis of the moon's orbit at the expense of Earth's rotation rate. The lunar and solar gravitational potentials, respectively, at an arbitrary point P on Earth's surface are

\[ \phi_m = \frac{-GM_m a^3}{2} \left( \frac{3 \cos^2 \psi_m - 1}{2} \right) \frac{1}{\xi_m^3} \]

(1)

\[ \phi_s = \frac{-GM_s a^3}{2} \left( \frac{3 \cos^2 \psi_s - 1}{2} \right) \frac{1}{\xi_s^3} \]

(2)

where \( \phi_m \) and \( \phi_s \) are the lunar and solar disturbance potentials, respectively. The equilibrium tide corresponding to \( \phi_m \) is raised semi-diurnally on a rotating Earth where the spin axis is normal to the plane of the moon's orbit. For the present-epoch Earth, with orbital obliquity of 23.5° ± 1.5°, the so-called tidal inequality introduces a strong latitude-dependent inequality (which vanishes on the equator) in the two semi-diurnal tides. The combination of lunar and solar tidal contributions considerably alters the tidal problem because of the diurnal inequality of the tides, which are generally not collinear.

Our major specific aim in the data analysis was to determine the number of neap-spring cycles per year. The year (the orbital period of Earth) is reasonably assumed to be invariant since the Precambrian. In that case, if a yearly (seasonal) period can be found, the power spectral density of the time sequence of neap-spring periods re-
veals the synodic neap-spring frequency $f_{ns}$. The corresponding sidereal period is $\tau_{sid} = 2/f_{ns} - 1$; the factor of 2 corresponds to the two neap-spring periods per lunar orbital period. The semimajor axis is given by

$$\xi = \left(\frac{\tau_{sid}}{2\pi}\right)^{2/3} [G(M_m + M_s)]^{1/3}$$

(6)

where eccentricity is ignored.

To capture the neap-spring cyclicity in the BCC records, we reconstructed the core time sequence with a singular spectrum analysis (SSA) algorithm (14) followed by computation of the periodogram (discrete Fourier transform (DFT)). We checked the results by direct counting of neap-spring interval thicknesses. Because SSA removes much of the noise component by eigenvalue selection for the reconstruction, the 2σ error bounds were also determined by maximum likelihood estimation (MLE) and the raw time sequence with initial frequency guesses from the DFT (15); statistical error bounds from MLE using raw data sequences contain the complete spectrum of noise as well as data and are the most meaningful measure of noise error (16). The BCC data yielded a semimajor axis of $3.45 \times 10^{10}$ cm. Corrections for apsidal rotation of the moon’s orbit and nodal regression are ignored because these corrections are of the same order as the computed random errors of the data.

Previous reports on the Elatina data (6) used the neap-spring (dark band) data. The calculation we report is based on MLE; our retreat rate is somewhat higher and is more consistent with the Apollo data (data gathered by means of the laser reflector deposited on the moon during the Apollo program) and the BCC data. The Elatina tidalites appear to be the most noise-free records available; MLE confirms that the noise is low.

The Pennsylvanian records (Mansfield and Pottsville) (Fig. 2) are more difficult to decode because the primary data are individual laminas and any yearly signal is of insufficient clarity to provide a yearly neap-spring count. Moreover, without an absolute (for example, annual) time reference, these data correspond only to the number of terrestial rotations per lunar orbital period; hence, the periods are lower bounds of the true lunar orbital period. The Mansfield periodogram (Fig. 3A) discloses a cluster of four lines, ranging in synodic frequency from 0.035 to 0.045 per lamina, which yield a spread of semimajor axis values. If the record of individual laminas is subject to erosion and loss of laminas, the least affected frequency (based on a scaled spectrum) is the lowest frequency. For the Mansfield Formation, this synodic frequency is 0.035 per lamina, which is the value used in the computation of the orbital curve of growth (OCG). Attempts to “tune” the spectrum to basic semiannual and annual tidal periods are not sufficiently viable; further work may yield an improved Pennsylvania estimate of the lunar orbital period.

To improve the signal/noise ratio of the Pottsville record, we reconstructed the time sequence with SSA (maximum autocovariance lag = 400; eigenvalue spectrum range from 1 to 12). Fig. 3B shows the DFT of the subsequent computation of the spectrum from the reconstruction. The long-period record consists of a remarkable sequence of relatively narrow lines, of which the two longest periods suggest annual and semianual lines with $-5$ to 10% loss of laminas. As in the case of the Mansfield data, attempts to tune the frequency scale of the spectrum have not yet been successful, and we use the same argument to select the spectral line at 0.035 per lamina as the least corrupted.

The linear least squares (LLS) “floating” fit (that is, a fit unconstrained by the Apollo data—derived retreat rate) uses BCC, Elatina, and Mansfield period estimates (Fig. 4). The least squares (LS) fit gives a mean retreat rate of 3.25 cm year$^{-1}$; it is influenced somewhat by the small error for the Mansfield data and more strongly by the small range estimate for age. For completeness, Fig. 4 also shows a linear extrapolation from the present retreat rate and semimajor axis of the moon’s orbit. It passes through

![Fig. 3. (A) Periodogram of the Mansfield synodic data, showing a neap-spring region cluster. The lunar orbital period corresponding to each peak (in days) is shown; the value of 28.3 (in parentheses) is the only member consistent with the Pottsville data. (B) DFT of SSA reconstruction of the Potts-ville data. The boxed area is the neighborhood of the expected neap-spring period. Other peaks can be identified (with periods ranging downward from annual) but appear to be corrupted by loss of laminas. Only the line at 0.035 per lamina provides a period consistent with the neap-spring period.](image)

![Fig. 4. OCG derived from lunar orbital period estimates from tidalite measurements. The modern value and estimates from Mansfield data (305 Ma, O), Pottsville data (312 Ma, ●), Elatina data (650 Ma), and BCC data (900 Ma) are shown. Error bars for Elatina and BCC (≥100 My) are not relevant to estimates of the lunar orbital period; these bars are abscent for the Pottsville and Mansfield data because of the small age uncertainty (≤5 My). The dashed line is a constant-slope datum using Apollo values for $\xi$ and $\Delta f/dt = 3.82$ cm year$^{-1}$. The dotted line is a LLS fit to the four data, unconstrained by the Apollo $\Delta f/dt$. The solid line is a second-order LS fit constrained by the Apollo $\Delta f/dt$.](image)

![Fig. 5. Sidereal lunar orbital period (in present-day) versus age. The 300-Ma value (Mansfield and Pottsville data) should be revised upward by a small but uncertain increment subject to an unknown absolute time reference.](image)

### Table 1. Tidalite-derived synodic lunar orbital periods.

<table>
<thead>
<tr>
<th>Formation</th>
<th>Period (days)</th>
<th>$+2\sigma$</th>
<th>$-2\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCC</td>
<td>25.0</td>
<td>25.3</td>
<td>24.7</td>
</tr>
<tr>
<td>Elatina</td>
<td>26.2</td>
<td>26.2</td>
<td>26.2</td>
</tr>
<tr>
<td>Pottsville</td>
<td>28.7</td>
<td>28.8</td>
<td>28.3</td>
</tr>
<tr>
<td>Mansfield</td>
<td>28.3</td>
<td>28.2</td>
<td>28.4</td>
</tr>
<tr>
<td>Modern</td>
<td>29.5</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
the Elatina-derived value of $\xi$ and just grasps the $2\sigma$ error estimate from the BCC data.

Calculation of the second-order LS OCG (Fig. 4) is constrained by the Apollo data-derived retreat rate, $\Delta d/dt = 3.82 \pm 0.07$ cm year$^{-1}$ (17), and more generally by the requirement that $\Delta d/dt \leq 0$ so long as angular momentum flows from Earth (or, equivalently, that $\Phi_{oc}$ corresponds to a tidally "leading" Earth). The value for $\xi$ is given for the present epoch (18). The response of the sun to the solar tide raised on Earth is to feed angular momentum from the sun into Earth's orbit. The sun's contribution to the Earth-moon problem can be handled as a perturbation to Earth's rotation and is therefore not included directly in the following discussion. However, the sun's tidal potential affects Earth markedly through friction and the LOD.

The Pottsville and Mansfield data (Fig. 4) lie $\approx 5\sigma$ to $6\sigma$ away from the regression lines. For the Pottsville, Mansfield, and Elatina data, the errors in age are too small to show as error bars. The Pottsville and Mansfield data may genuinely lie this far from the computed regression line; alternatively, they may be affected by a bias to smaller $\xi$ (from laminae erosion), a bias to larger $\xi$ (because of the lack of an absolute time base), or both. We have included these data because of the potential importance of the displacement from the OCG-computed variation of $\xi$ with time. The errors shown in Fig. 4 and discussed here are assumed normally distributed random. The possibility of significant (nonrandom) error from biases remains possible for the Pennsylvanian data.

Because the second-order LS fit (Fig. 4) is not strictly linear, the working hypothesis is that nonconstant tidal friction from the late Proterozoic to the present cannot be ruled out and requires a longer record of tides. Such a nonlinearity is consistent with the breakup of Pangaea and the development of shallow seas and higher friction in the Paleozoic. However, wave drag and the potential importance of oceanic resonances leave this matter unresolved (19). The sidereal lunar orbital periods for the BCC, Elatina, Mansfield, and modern geological times are shown in Fig. 5 (with $2\sigma$ error estimates and associated estimates of age uncertainties) and are listed in Table 1. If we assume that eccentricity can be ignored, the torque $T$ on the moon's orbit (ignoring the sun's contribution) is obtained by differentiating Eq. 3 with respect to $\Psi$,

$$T = \frac{3}{2} \left( \frac{GM_{\odot} a^5}{\xi^6} \right) k_i \sin(2\delta)$$

(7)

so that for constant $\xi$ (that is, constant $\Delta d/dt$),

$$\xi^{1/2} \sim k_i \sin(2\delta)$$

(8)

which implies a powerful increase in $k_i \sin(2\delta)$ with increasing $\xi$. Solving Eqs. 7 and 8 for $\delta$ yields

$$\delta \sim 0.43 \frac{\xi}{k_i}$$

(9)

for small values of $\delta$, where $\xi$ is in centimeters per year and $\delta$ is in degrees.

The lag angle is often assumed fixed. However, if Takeuchi's $k_i$ (see (19)) is held fixed versus time, which seems reasonable, then the lag angle must evolve. Figure 6 gives the lunitudal angle for 0.31 $\leq k_i$ $\leq$ 0.356 for the assumption of constant friction. The lag angle $\delta$ varies from 3.2° to 4.6° for the smallest value of $k_i$, and correspondingly for larger $k_i$. Corresponding quality factors $Q$ range from ~11 to 19; this range is compatible with earlier derivations from Holocene data. Generally, $\delta$ varies by $\approx 1.6^\circ$ independently of the Love number.

Munk and MacDonald (20) estimated the present rotational deceleration of Earth attributable only to the lunar tidal component to be $-4.81 \times 10^{-25}$ rad s$^{-2}$. The BCC (900 Ma) tidal data indicate that if only the lunar component is considered, the LOD was 19.2 hours during the late Proterozoic. If the solar component of 21% is added (assuming a common $k_i$ value), then the LOD was $\approx 18.2$ hours at that time (Table 2).

The energy of the lunar orbit with time increases (as does the major axis grows) at the expense of Earth's rotational energy. The effect is to lengthen the monthly cycle observed in the sedimentary record. In all, 4.12 $\pm$ 0.18 $\times 10^{15}$ ergs of terrestrial rotational energy was lost over the past 900 My. The gain in orbital energy of the moon during this time was 3.40 $\pm$ 0.03 $\times 10^{15}$ ergs. The difference appears as thermal (frictional) energy, where the generated heat $H_{\text{friction}} = 2.92 \times 10^{19}$ erg s$^{-1}$ (excluding any solar contribution) (21, 22). This value is $\approx 50\%$ of that estimated by Munk and MacDonald, but it is a mean over 900 My during which time, based on the increase in tidal coupling with time, the frictional loss would have varied. This value is $\approx 10\%$ that of the present-day mantle radioactive heat production ($8.5 \times 10^{12}$ cal s$^{-1}$ or $4.5 \times 10^{-12}$ cal g$^{-1}$ year$^{-1}$) (22). Table 2 gives a summary of parameters for BCC time (900 Ma).

**REFERENCES AND NOTES**

4. The equilibrium tide is defined in terms of an Earth that lacks land masses and has uniform ocean depth and instantaneous tidal propagation speed.
13. Both the moon and Earth move relative to the center.

![Fig. 6. Estimate of $k_i \sin(2\delta)$ versus age and $Q$. Calculations are based on second-order LS fit to data (constrained by the Apollo $d/dt$). In terms of Takeuchi's $k_i$ (20), $k^i = 0.256$ (density, 2.7 g cm$^{-3}$); $k^i = 0.281$ (density, 3.0 g cm$^{-3}$); $k^i = 0.29$ (23); and $k^i = 0.31$ (22).](image)

**Table 2.** Late Proterozoic (900 Ma) luni-lunet parameters for BCC data. Values in parentheses are incremented by 21% to include the solar contribution to tidal friction. The $+2\sigma$ error estimates are based only on internal Gaussian (normally distributed) noise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>$+2\sigma$</th>
<th>$-2\sigma$</th>
</tr>
</thead>
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<tr>
<td>Sidereal period (days)</td>
<td>23.4</td>
<td>23.6</td>
<td>23.1</td>
</tr>
<tr>
<td>Semimajor axis (cm)</td>
<td>3.453 $\times 10^{10}$</td>
<td>3.478 $\times 10^{10}$</td>
<td>3.429 $\times 10^{10}$</td>
</tr>
<tr>
<td>Orbital momentum (kg m$^2$ s$^{-1}$)</td>
<td>2.727 $\times 10^{11}$</td>
<td>2.737 $\times 10^{11}$</td>
<td>2.717 $\times 10^{11}$</td>
</tr>
<tr>
<td>Torque (dyn ecms)</td>
<td>9.600 $\times 10^{23}$</td>
<td>9.636 $\times 10^{23}$</td>
<td>9.567 $\times 10^{23}$</td>
</tr>
<tr>
<td>Orbital energy (ergs)</td>
<td>$-4.242 \times 10^{36}$</td>
<td>$-4.212 \times 10^{36}$</td>
<td>$-4.272 \times 10^{36}$</td>
</tr>
<tr>
<td>Length of day (hours)</td>
<td>19.2 (18.2)</td>
<td>19.0 (17.9)</td>
<td>19.5 (18.5)</td>
</tr>
<tr>
<td>Days per year</td>
<td>456 (481)</td>
<td>462 (489)</td>
<td>450 (473)</td>
</tr>
</tbody>
</table>
of mass in such a way as to conserve its position inertially. The present separation of Earth and moon is 3.564 x 10^10 m, 3.94 x 10^10 m, and 4.007 x 10^10 m [see (18)]. The BCC angular momentum is partitioned between the moon (h_moon) and Earth (h_Earth). Earth has 1.23% of the total system angular momentum, and the moon has the remaining 98.8%. Thus, the moon’s present-epoch share of orbital angular momentum must be reduced by 1.23%, the fraction belonging to Earth. For the earlier time periods, the barycenter shifts closer to Earth’s center. For the Earth-sun barycenter, the sun is taken as infinitely massive and the moon's mass is ignored. The obliquity of the lunar orbit to Earth’s equator is necessarily ignored here.


16. In MLE computations, we assume noise is Gaussian (normally distributed). Additionally, the frequency scale is based on near-spring laminae count (index) rather than on true frequency, errors attributable to lost laminae can contract the time scale.


21. Of the conservation laws for momentum and energy, the former is easier to deal with because momentum is partitioned primarily between terrestrial rotation and the lunar orbit, whereas energy is partitioned three ways, between these two parameters and frictional loss [W_L]. For constant f, dH/dt varies as t^1/2. Conversely, for constant f or approximately constant dH/dt (Fig. 3), f ~ t^{-1/2}. Moreover, for constant f, the ratio of k_L to Q (20) must be constant. It is difficult to assess the invariance of k_L over the past eon, and perhaps it has changed little, but the tidal lead angle has almost certainly not been fixed—and indeed has probably changed by a large factor—because the terrestrial gravity dispersion field caused by the presence of the moon varies as t^{-3}.


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