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Three-dimensional seismic refraction tomography: A comparison of two methods applied to data from the Faeroe Basin

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Abstract

This paper presents a comparison of two tomographic methods for determining three-dimensional (3-D) velocity structure from first-arrival travel time data. The first method is backprojection in which travel time residuals are distributed along their ray paths independently of all other rays. The second method is regularized inversion in which a combination of data misfit and model roughness is minimized to provide the smoothest model appropriate for the data errors. Both methods are nonlinear in that a starting model is required and new ray paths are calculated at each iteration. Travel times are calculated using an efficient implementation of an existing method for solving the eikonal equation by finite differencing. Both inverse methods are applied to 3-D ocean bottom seismometer (OBS) data collected in 1993 over the Faeroe Basin, consisting of 53,479 travel times recorded at 29 OBSs. This is one of the most densely spaced, large-scale, 3-D seismic refraction experiments to date. Different starting models and values for the free parameters of each tomographic method are tested. A new form of backprojection that converges more rapidly than similar methods compares favorably with regularized inversion, but the latter method provides a simpler model for little additional computational expense when applied to the Faeroe Basin data. Bounds on two model features are assessed using regularized inversion with combined smoothness and flatness constraints. An inversion of synthetic data corresponding to 100% data recovery from the real experiment shows a marked improvement in lateral resolution at deeper depths and demonstrates the potential of currently feasible 3-D refraction experiments to provide well-resolved, long-wavelength velocity models. The similarity of the final models derived from the two tomographic methods suggests that the results from the new form of backprojection can be relied on when limited computational resources rule out regularized inversion.

1. Introduction

Three-dimensional (3-D) seismic refraction surveys are still rare but are likely to become more common as the number of available land and marine recording units increases. Refraction surveys provide large-scale (macro) velocity models. Three-dimensional experiments are performed when a volume image as opposed to a cross-sectional image is needed or when there is no obvious "strike" direction. The density and uniformity of subsurface sampling in three dimensions is rarely as good as that which is achieved in most 2-D surveys, and this creates the greatest challenge in terms of producing reliable results when applying 3-D tomographic methods.

In this paper we consider the problem of determining 3-D isotropic velocity structure from first-arrival travel times. The times will normally correspond to refracted wave paths but could represent direct arrivals or diffractions. We assume there exists a reliable estimate of the observational uncertainties due to timing and picking error. We assume the uncertainties associated with the source and receiver positions and the source times to be negligible with respect to the travel time uncertainties. We are primarily concerned with the construction of the most featureless model that fits the data according to their uncertainties. We also seek other reasonable models that fit the data and the methods by which they are obtained since this places bounds on our solution.

We analyze data from the Faeroe Basin that is one of the most dense 3-D refraction data sets currently available. Over 50,000 refracted arrivals have been picked from 29 ocean bottom seismometers (OBSs), constraining the basin sediment and underlying basement velocities to ~10 km depth. These data provide an excellent opportunity to assess the imaging capability of 3-D refraction experiments and compare the performance of two popular 3-D tomographic schemes. Using real data avoids the use of unrealistic synthetic data or an unfair advantage for one method over another.

After a background section on refraction tomography, we present the technique used for travel time calculation, followed by a description of the two tomographic methods. We then describe the Faeroe Basin experiment and data, followed by an application of both tomographic methods to these data as well as related synthetic data sets. Our results include assessments of the performance of each tomographic method and of the experimental design, assuming there had been 100% data recovery.

2. Seismic Refraction Tomography

Examples of studies in which 3-D velocity structure has been obtained from refraction data include those by *Hammer et al.* [1994], *Toomey et al.* [1994], and *Zelt et al.* [1996]. The two former studies were of oceanic crust, the latter of continental crust. The method employed by *Hammer et al.* [1994] is characterized by a model parameterization that uses a series of continuous functions that is well suited to problems with sparse ray coverage; only 123 arrival times were available. *Toomey et al.* [1994] refer to their method as an "adaptive inverse modeling tool" owing to the relatively large amount of prior information incorporated into the inversion. The main features of Toomey et al.'s method are its jumping strategy and regularization in the form of vertical and horizontal smoothness constraints. A 200 m node spacing was used for their inversion in which ~130,000 model parameters were constrained by ~1500 travel times. *Zelt et al.* [1996] used a form of iterative backprojection tomography first described by *Hole* [1992] to define upper crustal velocity structure to ~15 km depth using P_g travel times. This method is characterized by the uniform distribution of slowness perturbations along ray paths and the smoothing applied to the model update at each iteration. Approximately 4000 travel times constrained ~500,000 model parameters on a uniform grid with a 1.2 km node spacing.

In this paper we apply two common but very distinct forms of tomography to the Faeroe Basin data: regularized inversion [e.g., *Toomey et al.,* 1994] and backprojection [e.g., *Hole,* 1992]. The methods have been altered as compared with their usual implementations to create two general purpose algorithms for refraction tomography. The free parameters associated with each method have been selected to make the final models similar and the simplest that fit the data. We have applied each method using as little prior information as possible, although we show how regularized inversion can be used to test particular model features by applying additional constraints.

The advantage of backprojection is its minimal computational and memory requirements and therefore the ability to use relatively fine model grids to allow high spatial resolution. The advantage of regularized inversion is the ability to include prior information, such as solution simplicity, so that it can provide the "minimum structure" model for a given level of fit to the data. According to the conventional viewpoint, the disadvantages of backprojection are its tendency to produce smeared results and its sensitivity to anisotropic ray coverage. The disadvantages of regularized inversion are its computational and memory requirements and the arbitrariness of the parameter values that determine the trade-off between data fit and the solution of the prior constraint equations.

3. Method of Forward Modeling

We use a modified form of *Vidale*'s [1990] method that calculates first-arrival travel times on a uniform grid by solving the eikonal equation by finite differencing. Times are calculated progressively away from a source on the sides of an "expanding" cube, one side being completed before the next is considered. Data with fewer receivers than sources are timed more efficiently in the reverse direction, treating each receiver as a source. In its original form, Vidale's method is not valid in the case of large velocity gradients or contrasts. *Hole and Zelt* [1995] present a two-step modification that corrects this problem: (1) the inclusion of head wave operators and (2) a reverse propagation through the model using a wall source initiated on one edge of the model. Ray paths are obtained by following the steepest gradient of the time field from a receiver back to the source [*Vidale*, 1988].

We have further modified the algorithm to improve its speed by a factor of 5 when applied to the Faeroe Basin data. The nodes comprising a side of the expanding cube are timed in an order according to the times associated with the nodes on the face directly behind the side to be timed, from fastest to slowest. For efficiency, a quick sort technique [*Press et al.,* 1992] is used. The order in which the sides of the expanding cube are timed is determined by the face that contains the node with the fastest time. In this way the expanding cube will track the shape of the true wave fronts as close as possible and thereby minimize the number of head wave operators and reverse propagations required.

The maximum picked arrival time for each source may vary because of the recording geometry or data quality. A computational saving is realized by terminating the propagation from a source once the nodes around all picked receivers have been timed. If a reverse propagation is required, the nodes involved are limited to those inside the same cube that was timed in the initial propagation. In addition, the reverse propagation need not begin at the edge of the model, or the edge of the cube timed in the initial propagation, but at the face farthest from the source that includes a node timed with a head wave operator. Similarly, the reverse propagation can be terminated before reaching the opposite side of the model or the cube timed in the initial propagation, if all nodes timed with a head wave operator in the initial propagation have been retimed and if no nodes are replaced with faster times for the current face being timed.

The bathymetry varies smoothly from about 1000 to 1500 m depth in the Faeroe Basin (Figure 1). The shallowest sediments of the basin are relatively low velocity (modeled as 1.6 km/s). The bathymetry is treated in two ways. First, for a particular (x, y) position the velocity of the node at a depth closest to the bathymetry equals the average of the true velocity over half the node spacing above and below the node. Second, to calculate the times at the nodes near a source (OBS), a straight ray approximation is used in a small cube around the source [*Vidale*, 1990] in which the velocity at the source is that of water when timing nodes that are above the source and that of sediment when timing nodes that are below the source.

4. Backprojection

We have used an iterative backprojection method based on that described by *Hole* [1992]. For the inverse step the model is parameterized in terms of constant-slowness cells, the cells being equal in size to the node spacing of the forward step. The slowness in each cell equals the average of the slownesses of the nodes at the eight corners of the cell. For each iteration, two sums are accumulated for each cell: (1) the ray-path-averaged slowness perturbation for each ray passing through the cell, equal to $\Delta t/l$, where Δt is the travel time residual (observed minus calculated time) of the ray and l is its total length, and (2) the number of rays passing through the cell. After all rays are traced, the slowness update for each cell through which at least one ray passes equals the first sum divided by the second sum. To stabilize the inversion, a moving average (MA) filter is applied to the slowness update before adding it to the current slowness model. The size of the MA operator is usually reduced with iteration, thereby stabilizing the inversion by defining progressively smaller wavelength model features with iteration. We refer to the full sequence of MA operators as the smoothing schedule. After each iteration the model is reparameterized from the cell-based to the node-based format by calculating the average slowness of the eight cells surrounding each node. Travel times and ray paths are recalculated through the updated model, and iterations are continued until a normalized data misfit χ^2 of one is achieved, or the misfit is not significantly improved from the previous iteration.

We now describe our modifications to the basic algorithm. *Toomey et al.* [1994] point out that the inverse relationship between slowness and velocity means that an equal distribution of the slowness perturbation along a ray path will bias the final model toward increased heterogeneity in high-velocity regions. To remove this bias, we apply a nonuniform backprojection scheme whereby each cell receives a slowness perturbation proportional to the cell's prior (i.e., starting) slowness. For a given ray path with an associated travel time residual of Δt passing through M cells, the equation relating the slowness update in each cell to Δt is

$$\Delta t = \sum_{j=1}^{M} l_j \Delta s_j \tag{1}$$

where l_j and Δs_j are the ray path length and slowness perturbation, respectively, in the jth cell. For $\Delta s_j \propto s_j$, where s_j is the cell's prior slowness, solving (1) for the slowness update for the jth cell leads to

$$\Delta s_j = \frac{s_j \Delta t}{\sum_{k=1}^{M} l_k s_k} = \frac{s_j \Delta t}{t^{\mathrm{pre}}} \tag{2}$$

where t^{pre} is the predicted travel time for the current model.

The second modification weights the slowness perturbation in (2) by the inverse of the pick uncertainty corresponding to the observed travel time for the ray. Therefore the following two sums are accumulated for each cell: (1) $\Delta s_j/\sigma$ for each ray passing through the cell where Δs_j is given by (2) and σ is the uncertainty assigned to the observed data for that ray, and (2) $1/\sigma$ for each ray passing through the cell. As before, the slowness update equals the first sum divided by the second sum for each cell.

The third modification arises from the fact that a ray may pass though a region of the model for which the true velocity structure is known or is assumed to be known, as is the case in a layer- stripping approach. To account for this, the summation in (2) is restricted to cells for which the slowness is to be solved for, and the number of cells is replaced by the effective number of cells M^e. Since the water velocity is known for the Faeroe Basin, the slowness update is restricted to the cells below the bathymetry. The slowness update Δs_j for the jth cell becomes

$$\Delta s_j = \frac{\sum_{i=1}^{M_j} \frac{1}{\sigma_i} \left[\frac{s_j \Delta t_i}{t_i^{\mathrm{pre},e}} \right]}{\sum_{i=1}^{M_j} \frac{1}{\sigma_i}} \tag{3}$$

where M_j equals the number of rays sampling the jth cell; Δt_i and σ_i equal the travel time residual and uncertainty, respectively, of the ith ray; and $t_i^{\mathrm{pre},e}$ equals the predicted travel time over the effective cells for the ith ray. No matrix is inverted in this procedure, and the memory required in addition to the forward step is two arrays the size of the model.

Backprojection suffers from slow convergence owing to two factors: (1) backprojection is roughly equivalent to an inversion in which only the diagonal terms of the matrix $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ are retained, where $\mathbf{L}$ is the partial derivative matrix with elements l_{ij} equal to the length of the ith ray in the jth cell [*Humphreys and Clayton,* 1988], and (2) smoothing is applied to the slowness update at each iteration. To accelerate the convergence, we present a modification we call scaled backprojection based on a procedure described by *Humphreys and Clayton* [1988]. We assume the slowness update given by (3) after it is smoothed provides the correct direction in model space, but the length of the update is solved for by determining a multiplicative scalar α such that the weighted data misfit ϕ is minimized. Therefore we minimize the function

$$\begin{aligned} \phi &= \sum_{i=1}^{N} \left[\frac{\Delta t_i}{\sigma_i} \right]^2 = \sum_{i=1}^{N} \left[\frac{t_i^{\mathrm{obs}} - \sum_{j=1}^{M} l_{ij} s_j^k}{\sigma_i} \right]^2 \\ &= \sum_{i=1}^{N} \left[\frac{t_i^{\mathrm{obs}} - \sum_{j=1}^{M} l_{ij} (s_j^{k-1} + \alpha \Delta s_j)}{\sigma_i} \right]^2 \end{aligned}$$

with respect to α, where there are N observed data and M model cells, t_i^{obs} is the ith observed travel time with an assigned error of σ_i, s_j^k is the slowness of the jth cell for the kth iteration, and Δs_j is given by (3). This leads to

$$\sum_{i=1}^{N} \left[\frac{t_i^{\mathrm{obs}} - \sum\limits_{j=1}^{M} l_{ij} s_j^{k-1} - \alpha \sum\limits_{j=1}^{M} l_{ij} \Delta s_j}{\sigma_i} \right] \left[\frac{\sum\limits_{j=1}^{M} l_{ij} \Delta s_j}{\sigma_i} \right] = 0. \tag{4}$$

The sum $\sum_{j=1}^{M} l_{ij} s_j^{k-1}$ is equal to the ith predicted travel time t_i^{pre} for the current iteration, and we define the sum $\sum_{j=1}^{M} l_{ij} \Delta s_j$ to be the ith predicted travel time residual $\Delta t_i^{\mathrm{pre}}$, i.e., the travel time perturbation associated with the slowness perturbation given by (3) using the ray paths of the current iteration. Solving (4) for α gives

$$\alpha = \frac{\sum_{i=1}^{N} \left[\frac{t_i^{\mathrm{obs}} - t_i^{\mathrm{pre}}}{\sigma_i}\right]\left[\frac{\Delta t_i^{\mathrm{pre}}}{\sigma_i}\right]}{\sum_{i=1}^{N} \left[\frac{\Delta t_i^{\mathrm{pre}}}{\sigma_i}\right]^2}.$$

Defining $\Delta t_i^{\mathrm{obs}} = (t_i^{\mathrm{obs}} - t_i^{\mathrm{pre}})$ to be the travel time residual for the current iteration and using vector notation so that, for example, the ith element of $\mathbf{\Delta t}^{\mathrm{obs}}$ equals $\Delta t_i^{\mathrm{obs}}/\sigma_i$, we obtain

$$\alpha = \frac{\mathbf{\Delta t}^{\mathrm{obs}} \cdot \mathbf{\Delta t}^{\mathrm{pre}}}{\mathbf{\Delta t}^{\mathrm{pre}} \cdot \mathbf{\Delta t}^{\mathrm{pre}}}. \qquad (5)$$

The vector $\mathbf{\Delta t}^{\mathrm{obs}}$ is known after the basic backprojection step, and $\mathbf{\Delta t}^{\mathrm{pre}}$ is obtained by either recalculating or recalling the ray paths for the current iteration. Division by zero in (5) is not a concern using real data with noise.

All tests we have performed using scaled backprojection on real and synthetic data show that it provides essentially the same results as using (3) alone, but in roughly one third the number of iterations. All backprojection results we present use scaled backprojection unless otherwise stated.

Typical results from backprojection are rough in comparison to the results from most other inverse methods for a given level of fit to the data. This is because when a rough slowness perturbation is added to the model, there may be no chance to smooth out this structure in subsequent iterations. This suggests the need to apply smoothing to the whole model or the total perturbation. We tested three schemes: (1) smoothing the whole model after each iteration instead of smoothing the perturbation, (2) smoothing the whole model after the data have been fit and then re-iterating a few times to fit the data again, and (3) smoothing the whole model a small amount after each iteration in addition to smoothing the perturbation. All three methods were tested and work about equally well in practice, but the third scheme was chosen since it represents the smallest change to the method described thus far.

5. Regularized Inversion

Regularization is an approach by which constraints, in addition to the data, are applied to an inverse problem to treat the underdetermined part of the solution [e.g., *Scales et al.,* 1990; *Phillips and Fehler,* 1991]. Usually the constraints result in the final model satisfying some property in addition to fitting the data. This property is often chosen so that the model has "minimum structure" since we seek models that include only structure that is required to fit the data according to its noise level [*Scales et al.,* 1990]. Minimum structure is usually measured in terms of model roughness, i.e., second spatial derivative [e.g., *Lees and Crosson,* 1989; *Parsons et al.,* 1996]. In this case, regularized inversion minimizes an objective function that includes norms that measure model roughness and data misfit. A tradeoff parameter is selected that provides the model with the least structure for a given data misfit. The final model for a nonlinear problem is assumed to be minimum structure if the starting model is "close" to the final model.

For ray-based travel time tomography it makes sense to minimize model roughness for several reasons: (1) infinite-frequency ray methods are valid for smooth media only, (2) travel times constrain only the long-wavelength model features since the data represent integrals through the model, (3) the linearization assumption of stationary ray paths is more likely to be satisfied for smooth models, and (4) a consideration of the first Fresnel zone means our recovered model should represent a smoothed version of the true structure.

We have used a jumping strategy [*Shaw and Orcutt,* 1985] that penalizes total model roughness. The objective function Φ minimized at each iteration is

$$\Phi(\mathbf{m}) = \delta\mathbf{t}^{\mathrm{T}}\mathbf{C}_d^{-1}\delta\mathbf{t} + \lambda[\mathbf{m}^{\mathrm{T}}\mathbf{C}_h^{-1}\mathbf{m} + s_z\mathbf{m}^{\mathrm{T}}\mathbf{C}_v^{-1}\mathbf{m}]$$

where $\mathbf{m}$ is the model vector; δt is the data residual vector; $\mathbf{C}_d$ is the data covariance matrix; $\mathbf{C}_h$ and $\mathbf{C}_v$ are the horizontal and vertical roughening matrices, respectively; λ is the trade-off parameter; and s_z determines the relative importance of maintaining vertical versus horizontal model smoothness. This leads to the system of equations

$$\begin{bmatrix} \mathbf{C}_d^{-1/2}\mathbf{L} \\ \lambda\mathbf{C}_h \\ s_z\lambda\mathbf{C}_v \end{bmatrix} \delta\mathbf{m} = \begin{bmatrix} \mathbf{C}_d^{-1/2}\delta\mathbf{t} \\ -\lambda\mathbf{C}_h\mathbf{m}_0 \\ -s_z\lambda\mathbf{C}_v\mathbf{m}_0 \end{bmatrix} \qquad (6)$$

where $\mathbf{L}$ is the partial derivative matrix with elements as defined earlier, $\mathbf{m}_0$ is the current model, $\delta\mathbf{m}$ is the model perturbation, and $\mathbf{m} = \mathbf{m}_0 + \delta\mathbf{m}$. This system is solved using the LSQR variant of the conjugate gradient algorithm [*Paige and Saunders,* 1982; *Nolet,* 1987].

$\mathbf{C}_d$ is a diagonal matrix with elements σ_{ii}^2, where σ_{ii} is the prior uncertainty of the ith travel time. The roughening matrices contain the 2-D and 1-D second-derivative finite difference operators that measure the model roughness in the horizontal and vertical directions. Each row of $\mathbf{C}_h$ contains the five nonzero elements of the Laplacian operator equal to $1/s_j$, $1/s_j$, $-4/s_j$, $1/s_j$, and $1/s_j$, where the elements correspond to a central cell and the four adjacent cells in the x and y directions and s_j is the prior slowness of the center cell. Similarly, each row of $\mathbf{C}_v$ has three nonzero elements equal to $1/s_j$, $-2/s_j$, and $1/s_j$, where the elements correspond to three adjacent cells in the z direction. The normalization by the prior slowness is applied to avoid a bias toward greater levels of model roughness in regions of high velocity.

Regularization accounts for a large number of the linear equations in (6); however, for the Faeroe Basin data, 99.95% of the matrix elements are zero and (6) can be solved rapidly (see appendix). Part of this speed is due to the fact that it is only necessary to apply a few iterations of LSQR to a nonlinear problem since the linearization is itself an approximation. We applied 50 iterations of LSQR for all results presented here and most final models required three or four nonlinear iterations of the inverse method, where the latter type of iteration refers to a forward calculation and a model update. Iteration herein refers to a nonlinear iteration.

According to (6), there are two free parameters, λ and s_z. However, s_z is held fixed for all iterations leading to a final model, whereas a range of λ values are tested at each iteration and that which gives the smallest data residual $\delta\mathbf{t}^{\mathrm{T}}\mathbf{C}_d^{-1}\delta\mathbf{t}$ is selected; for the last iteration we determine the value of λ that gives a normalized misfit of 1. This makes s_z the only free parameter.

6. Faeroe Basin Experiment and Data

In 1993, Mobil North Sea Limited contracted Hamburg University to acquire 2-D and 3-D OBS data over the Faeroe Basin to supplement conventional reflection data [*Zelt and Barton,* 1994; *Hughes et al.,* 1997]. The 3-D data represent one of the most dense networks of crustal refraction data recorded to date. Forty OBSs were deployed in a regular grid with a 6 km spacing (Figure 1). Two 60 liter air guns were fired every minute, providing a nominal shot spacing of 120 m and a total of 3800 shots along 11 intersecting profiles, for a total of just over 150,000 seismic traces. Only first arrival times from vertical component seismometers will be considered here. The shot and deployed OBS locations were determined using the Global Positioning System, providing a nominal accuracy of 30 m. Bathymetry recorded at each shot location varies smoothly between 1.0 and 1.5 km (Figure 1).

The usable data from 29 OBSs have a dominant frequency of 4-6 Hz, implying a wavelength of 0.25-1.6 km for P wave velocities of 1.5-6.5 km/s. There are 48 in-line and 271 broadside common-receiver profiles (Figure 2). The direct (water) wave is the first arrival to $\sim$5 km offset; at offsets between 5 and 10 km the apparent velocity of the first arrival generally increases from less than 2 to 4 km/s; at offsets between 10 and 20 km, apparent velocities are 4-4.5 km/s; and at offsets greater than 20 km the apparent velocity is 5-6 km/s. This suggests (1) the sea bottom is "soft"; (2) the basin sediments have velocities of less than 2 km/s, increasing with depth to $\sim$4.5 km/s; and (3) the arrivals at greatest offset correspond to waves sampling the underlying basement.

An examination of travel time reciprocity and first arrival times at profile intersection points allows precise OBS clock drift rates to be determined and suggests an overall timing accuracy of $\sim$40 ms and a final OBS positioning accuracy after relocation of 50 m. A total of 53,479 first arrivals were picked (Figure 3). Pick uncertainties were estimated using an automated scheme based on an empirical relationship between pick uncertainty and signal-to-noise ratio (SNR) [*Zelt and Forsyth,* 1994]. The average SNR is 7.2, and the average uncertainty is 56 ms, ranging between 40 and 100 ms.

7. Starting Models and Test Models

The selection of the starting model is important because the assumption of the linearization is that small perturbations to the starting model are determined. Using trial-and-error forward modeling and a least squares inverse method [*Zelt and Smith,* 1992], a four-layer, 1-D, linear-gradient velocity model was constructed that fits the average of the observed travel times calculated in 1 km offset bins. The top layer is 1.25 km thick and represents the average water column in the survey area. Below this, the three layers correspond to Tertiary sediments, Cretaceous sediments, and the crystalline basement [*Hughes et al.,* 1997]. The velocities at or near the top of these layers (1.6, 4.0, and 5.5 km/s) are constrained by the binned travel time data, but estimates for the vertical velocity gradients were obtained from amplitude modeling of the Faeroe Basin refraction data and nearby well data [*Hughes et al.,* 1997].

Different values of the smoothing parameters (s_z for regularized inversion and MA operator size for backprojection) were tested to establish the appropriate range of values for each method. All runs resulted in final models that were significantly biased at certain depths with respect to the starting model, primarily because the smoothing broadens the layer boundaries into transition zones. The 1-D average of these preliminary final models was selected as the starting model (Figure 4a). The results of inversions using this starting model yield a final model that is not significantly biased from the starting model at any depth (Figure 4b).

An alternate 1-D starting model (Figure 4a) was constructed that is less consistent with the amplitude, regional reflection, and well data [*Hughes et al.,* 1997] but provides a similar fit to the binned data in Figure 3 and results in unbiased final models. The larger vertical gradient in the second sedimentary layer of the alternate model affects the ray coverage in the layer (Figure 5). The alternate starting model is used to assess the inherently poorly constrained depth range of 6.5- 9.0 km in the preferred starting model due to a lack of ray bottoming points.

The preferred and alternate starting models provide initial root-mean-square (RMS) travel time misfits of 155 and 160 ms (not including the direct arrivals). The known bathymetry was incorporated into the starting models using a constant velocity (1.6 km/s) along the bathymetric surface and a laterally varying vertical velocity gradient below such that the velocity at the base of the first sediment layer at 4 km is 4 km/s. The first arrival times do not constrain the velocity in the upper 250 m of sediment (Figure 5). Given that the first change in slope of the first arrivals after the direct wave is, on average, 1.7 km/s, a velocity of 1.6 km/s was assigned to the top of the first sediment layer based on an extrapolation of the layer's gradient.

To assess lateral model resolution, a "checkerboard" test [e.g., *Hearn and Ni,* 1994] was performed in which the known model consists of the preferred starting model with the addition of a laterally alternating anomaly pattern of positive and negative squares extending from 2 km depth to the bottom of the model. The real data's source-receiver geometry was used, and Gaussian noise was added with a standard deviation equal to the pick uncertainties. The data were inverted using the preferred starting model. By assessing the recovered anomaly pattern, an estimate of the model's ability to resolve anomalies with a lateral dimension equal to the size of the anomaly pattern is obtained throughout the model. Only where there is good ray coverage will the recovered anomaly pattern resemble the true pattern since the horizontal smoothing included in both tomographic methods will otherwise yield a smooth, nonalternating model. We have used models with a 6×6 km ±0.2 km/s anomaly pattern.

Synthetic data calculated from the preferred final model for each method were also inverted. This is a check on the preferred final models since any structure not present in the final models from inverting the synthetic data cannot be resolved. As with the checkerboard test, the true source-receiver geometry and noise level were used, along with the preferred starting model.

8. Results

For all forward calculations the model is defined on a uniform 0.4 km grid extending from -2 to 46 km in the x direction, -2 to 28 km in the y direction (Figure 1), and 0 to 14 km in the z direction. A uniform 0.4 km cell size was used in the inverse step of the backprojection method, resulting in 315,000 independent model parameters. A horizontal cell size of 1.2 km was used in the inverse step of the regularized inversion method to reduce the memory requirements (see appendix), resulting in 35,000 independent model parameters. Using a finer grid size for backprojection is in keeping with its main advantages over regularized inversion as described in section 2.

To quantify the amount of structure in a final model, we calculate the L_2 norm of its horizontal roughness R and the perturbation of the final model with respect to the background (i.e., starting) model P.

$$R = \sqrt{\frac{1}{M^s}\sum_{j=1}^{M^s}\left[\frac{4v_j - v_j^{x-1} - v_j^{y-1} - v_j^{x+1} - v_j^{y+1}}{v_j}\right]^2},$$

$$P = \sqrt{\frac{1}{M^s}\sum_{j=1}^{M^s}\left[\frac{v_j - v_j^b}{v_j^b}\right]^2} \tag{7}$$

where M^s is the number of nodes sampled; v_j is the velocity of the jth node; v_j^{x-1}, v_j^{y-1}, v_j^{x+1}, and v_j^{y+1} correspond to the velocities of nodes spatially on either side of the jth node in the x and y directions; and v_j^b is the velocity of the jth node in the background model.

8.1. Preferred Starting Model

Using the preferred starting model, we determined a preferred final model by testing a range of free parameter values: s_z for regularized inversion and the smoothing schedule for iterative backprojection (Plate 1 and Figure 6). For regularized inversion, values of s_z from 0.05 to 1.0 were tested and minimums in P and R occur at s_z =0.1 (Table 1). Values of s_z greater than 0.25 result in so much vertical smoothing that the second and third sedimentary layers appear as one layer (Figure 4a). This suggests acceptable values of s_z between 0.1 and 0.25. The RMS difference between the two extreme final models in this range is 0.05 km/s. We selected a value of s_z =0.125 to produce the preferred final models since its 1-D average is closest to the starting model (Figure 4b).

The final model provides an RMS misfit of 53 ms (Figure 7) or a normalized misfit χ^2 of 1. The residual plots for all final models obtained from inverting the real data we present in this paper appear similar to that shown in Figure 7b, and all provide a normalized misfit of 1. There is a sharp drop in ray coverage below 6 km depth and a centrally located relative high at 10 km depth corresponding to rays turning at the top of the basement (Figure 8a). The greatest difference between final models obtained using other values of s_z occurs at z =6.8 km (Plates 2a and 2b), a depth range that is relatively poorly constrained by the data as described later. All runs required three iterations to fit the data.

For backprojection, several runs were made using a constant MA operator size for all iterations. These tests showed that a maximum lateral operator size of 4.4 km can be used to fit the data, however, starting with a larger operator size and reducing it to 4.4 km with iteration results in smoother final models. The preferred final model (Plate 1b and Figure 6b) was obtained by starting with a lateral operator size of 10 km and reducing it by 0.8 km with each iteration. The vertical operator size was chosen so that the 1-D average of the final model was as close as possible to the starting model (Figure 4b): 2 km for the first six iterations and 1.2 km for the remaining iterations. The ray coverage in the preferred final model is similar to that in the preferred final model obtained using regularized inversion (Figure 8).

Using a constant operator size of 4.4×4.4×1.2 km, it took 11 iterations to fit the data compared with nine iterations to obtain the preferred final model. This alternate final model has less perturbation but is rougher than the preferred final model (Plate 3a and Table 1). The final model obtained using *Hole*'s [1992] backprojection method and the same smoothing schedule as used to obtain the preferred final model, except extended by a factor of 3 to cover the 31 iterations required, is considerably rougher than the preferred final model (Plate 3b and Table 1).

8.2. Resolution Analysis

For the checkerboard test the free parameters chosen for each method were the same as those used to obtain the preferred final models. The results for both methods are good and similar to 6 km depth (Figure 9), below which the results are degraded as expected owing to poor ray coverage (Figure 8). Nevertheless, the recovered anomaly pattern is of the correct sign for a large central portion of the models below 6 km. The poor resolution between 6.5 and 9.0 km in both models corresponds to the lower half of the second sediment layer (Figure 4a) within which few rays have bottoming points (Figures 5a and 8).

To quantitatively assess the results of the checkerboard test, the semblance between the exact and recovered checkerboard models has been calculated using a 6×6×1.2 km operator centered at each node of the models (Figure 10). We refer to the obtained semblance values as the "resolvability" since it indicates the ability of a final model to resolve features of a particular length scale. Comparing Figures 9 and 10, resolvabilities above about 0.7 indicate well-resolved regions of the models. The resolvability of the final models is similar; however, it is slightly lower at some depths in the model from regularized inversion. This is likely due to the tendency for regularized inversion to provide a smoother model for a given level of fit to the data and the smaller cell size used in the inverse step for backprojection.

8.3. Alternate Starting Model

The relatively few number of rays with bottoming points between 6.5 and 9.0 km depth (Figures 5a and 8) suggests there will be poor constraint in this depth range, as confirmed by the checkerboard test. By using the alternate starting model, which has higher velocities and better ray coverage in this depth range as a result of a larger vertical gradient in the second sedimentary layer than the preferred starting model (Figures 4a and 5b), we test the sensitivity of the results to the velocity structure in this poorly constrained region. The free parameters for each method were the same as those used to obtain the preferred final models.

The final models for each method contain more structure in the 6.5-9.0 km depth range (Plates 2c and 3c) than the preferred final models (Plate 1) but, otherwise, are very similar. The magnitude of the anomalies between 4 and 6 km depth is slightly less than in the preferred final models. The perturbations of these final models with respect to the alternate starting model are 12% and 8% greater compared with the preferred final models for regularized inversion and backprojection (Table 1). The roughnesses of the final models are 7% more and 2% less for the regularized inversion and backprojection methods, respectively, compared with the preferred final models (Table 1). The least perturbed and smoothest of the four final models is that obtained by regularized inversion using the preferred starting model.

Although the enhanced structure between 6.5 and 9.0 km relative to the preferred final model satisfies the data, it represents structure not required by the data. This illustrates the influence of the starting model in the poorly constrained depth range and the need to choose the starting model carefully.

8.4. Inversion of Synthetic Data

Synthetic data calculated using the preferred final model for each method were inverted. The free parameters for each method were the same as those used to obtain the preferred final models. Any feature of the preferred final models that is not present in the recovered models cannot be resolved. A comparison of the recovered models (Plates 2d and 3d) with the preferred final models (Plate 1) shows close agreement of all major features for both methods. The preferred backprojection model contains more unresolved structure since the L_2 norm of the relative difference with the preferred final models is about 0.4% for backprojection and 0.3% for regularized inversion (Table 1).

8.5. Other Tests

A subset of the real data composed of every tenth pick was inverted to assess the sensitivity of each method to sparser ray coverage. The resulting 5335 travel times were inverted using the same free parameters for each method that were used to obtain the preferred final models. The maximum difference between the preferred and recovered models is 0.06 and 0.20 km/s for regularized inversion and backprojection, respectively. The L_2 norm of the relative difference for each method is small, about 0.2% and 0.3% for regularized inversion and backprojection, respectively (Table 1). Thus both methods are relatively insensitive to the sparser (1.2 km) shot spacing, although backprojection is slightly more so. This is not surprising, given the data's noise level and therefore the length scales of the features resolved in the preferred final models. The true shot spacing of 120 m was necessary to provide the phase correlation needed to pick the data and would also have been required for the best inversion results, had the data's timing errors been less.

We have seen that the constraint on the lower half of the second sedimentary layer between 6.5 and 9.0 km depth is poor. However, the preferred final models contain lateral variations in this depth range, up to 0.27 km/s for regularized inversion and 0.49 km/s for backprojection (Plate 1). As one end-member, it is instructive to construct a model that fits the data but contains no lateral variation in this depth range. We have used a simple extension of the regularized inversion method to accomplish this. Instead of applying a lateral smoothness constraint, a flatness constraint (first spatial derivative) is applied within the depth range of interest. This adds another free parameter to the inversion that is selected by trial and error, so that the final model contains essentially no structure between 6.5 and 9.0 km but remains as laterally smooth as the preferred final model elsewhere.

The resulting model has no more than 0.1 km/s lateral variation between 6.5 and 9.0 km (Plate 4a). Elsewhere, the main difference with respect to the preferred final model is an increase in the magnitude of the anomalies below 9 km depth by about 0.2 km/s. Overall, this model has slightly less perturbation than the preferred final model, but it is slightly rougher (Table 1). This model fits the data equally well and therefore suggests that lateral variation in the 6.5-9.0 km depth range is not required to fit the data, and the magnitude of the anomalies in the preferred final model below 9 km may be underestimated by as much as 0.2 km/s.

The anomaly with the largest magnitude (0.7 km/s) in the preferred final model is the central positive anomaly at 10 km depth (Plate 1). Relatively few ray paths constrain this feature (Figure 8) and so we have tested whether it is required by using a flatness constraint as described above, except between 9 and 14 km depth. The resulting model has no more than 0.1 km/s lateral variation between 9 and 14 km (Plate 4b). The magnitudes of the anomalies between 6.5 and 9.0 km have been significantly enlarged relative to the preferred final model, and overall, this model is rougher than the preferred final model but has slightly less perturbation (Table 1). This suggests that the anomalies below 9 km depth are not required to fit the data, but the alternative is a model with more structure in the poorly constrained depth range.

The final test we present is a synthetic study to show what the resolution of the Faeroe Basin data could have been if there had been no OBS losses or malfunctions and it were possible to pick a first arrival for all shots. This yields 150,480 travel times from 40 OBSs. A checkerboard test was performed using regularized inversion and the ray geometry corresponding to these data as described earlier for the actual picked data. The resolution below 6 km is seen to be much improved (Figure 11) over the result for the actual data (Figure 10) owing to the addition of many long-offset, deep-penetrating ray paths in the synthetic test. The resolution of 6 km features is good everywhere there is ray coverage.

9. Discussion and Conclusions

We have compared two first-arrival travel time tomographic methods for obtaining 3-D velocity structure: backprojection and regularized inversion. The goal of our modeling approach was to obtain the minimum-structure model, i.e., a model that contains only structure required by the data. The preferred final models from regularized inversion and a backprojection scheme that includes a small amount of whole model smoothing are similar (Plate 1 and Figure 6). Given the relatively large data uncertainties and sparse ray coverage below 6 km depth, models with a wide range of structure in the lower half of the model can satisfactorily fit the data. Testing a range of free parameters for both methods establishes bounds on the model parameters. The maximum difference between five final models was calculated (Figure 12): the two preferred final models and the final models in Plates 2a, 2b and 3a. In most regions the maximum difference is less than 0.2 km/s, but it is up to 0.4 km/s in two locations: (1) near the anomaly with the greatest magnitude at 10 km depth and (2) in the lower half of the second sediment layer, a poorly constrained depth range due to the absence of bottoming rays (Figure 5a).

Constant-velocity surfaces from the preferred final models were compared with isopach and structural maps obtained by *Mudge and Rashid* [1987] and *Rumph et al.* [1993] from regional seismic reflection profiles. The 4.0 km/s surface at 4-5 km depth generally corresponds to the base of the Paleocene correlated by *Hughes et al.* [1997], and the 5.8 km/s surface at 9-11 km depth corresponds to the basement, including a major basement high that is expressed in the preferred final velocity model as the largest magnitude anomaly (+0.7 km/s) at 10 km depth (Plate 1).

9.1. Performance of Tomographic Methods

The basic backprojection method produces comparatively rough models (Plate 3b) and converges slowly, but scaled backprojection including whole model smoothing yields smoother models and converges about 3 times faster. In all tests we performed, the scaled form of backprojection provided essentially the same results as the equivalent unscaled run. For the model smoothing we applied, values of the scale factor α given by (5) were typically between 2 and 4, explaining the factor of 3 reduction in the number of iterations. The amount of whole model smoothing we have used is small in comparison to the smoothing of the slowness perturbation at each iteration and is therefore not a free parameter in the sense that the results are nearly insensitive to its value. Whole model smoothing means that even with the changing position of ray paths with iteration, a rough model feature can be removed in subsequent iterations.

Regularized inversion can provide slightly simpler models for relatively little added computational expense in the case of the Faeroe Basin data (see appendix). Although using a smaller grid size in the inverse step for regularized inversion was not warranted for the Faeroe Basin data, it may have been if the data's SNR had been higher. In this case and for larger data sets, the added expense will grow quickly and may limit the user to backprojection. Our results have shown that backprojection can perform nearly as well.

Regularized inversion provides a simple way to assess model features and perform hypothesis testing [e.g., *Toomey et al.,* 1994]. Localized flatness constraints were applied to place reasonable bounds on two model features (Plate 4). By forcing the model in certain "directions," nonuniqueness, parameter bounds, and trade-offs can be established. It would be less straightforward to accomplish this with backprojection.

Although smearing along ray paths is often observed with backprojection methods, this has not been a problem in this study. This may be due to (1) a more isotropic ray coverage that occurs in 3-D problems relative to 2-D problems in which all ray paths lie in a single plane, (2) the level of smoothing and coarseness of the model parameterization, and (3) starting models that are relatively close to the final models near the OBSs (Plate 1). The smoothing associated with both methods acts to smear out anomalies to an extent determined by the local ray sampling, and backprojection has not performed significantly differently than regularized inversion in this regard. An inversion of every tenth pick of the Faeroe Basin data showed that backprojection is only slightly more sensitive to sparser ray coverage.

The result of using an alternate starting model that differs from the preferred starting model in a poorly constrained depth range was a final model that differed from the preferred final model, primarily in the same depth range. This result, along with the results of using flatness constraints (Plate 4), demonstrates that the data can be fit using a relatively wide range of structures in the lower half of the model. However, when including as little prior information as possible, both methods, to varying degrees, have produced preferred final models that overall are the most featureless of any final models we obtained that fit the data.

9.2. Free Parameter Selection

Regularized inversion produced a slightly simpler final model than backprojection for the same level of fit to the data, but perhaps more importantly, the selection of the free parameter is more straightforward with regularized inversion. The free parameter for regularized inversion is s_z, the relative weighting of vertical versus horizontal roughness. A suitable value is determined by testing relatively few values since the corresponding effect is predictable.

For backprojection the free parameter is the smoothing schedule, i.e., the size of the MA operator at each iteration. Using a constant operator size for each iteration and testing various sizes is not sufficient since beginning with a relatively large operator size and decreasing it with iteration results in smoother final models. We used the following approach to produce the simplest final models. A maximum operator size was determined that would allow the data to be fit to within their uncertainties using a constant operator size for all iterations. An initial operator size was then chosen that was significantly larger (2 to 3 times), and it was decreased slightly with iteration until it reached the maximum operator size. The results are not sensitive to the initial operator size but will affect the number of iterations required.

Using a large MA operator keeps the model smooth; however, a smaller operator generally reduces the number of iterations required to fit the data. This is advantageous since the magnitude of anomalies in poorly constrained regions of the model often "blow up" with iteration. The conflicting advantages of large and small operator sizes requires trial-and-error testing to determine the best rate at which to decrease the MA operator size in the smoothing schedule, and there appears to be no well-defined set of tests to perform. Regularized inversion does not suffer from this problem since the smoothing is formally included in the minimization process. However, a potential pitfall with regularized inversion is taking too large a step in model space that may violate the linearization assumption; this can be tested by choosing a larger value of λ at each iteration than that which provides the best fit to the data and comparing final model roughness and perturbation. Backprojection algorithms that would mimic the smoothing associated with regularized inversion by using a spatially varying smoothing operator based on ray coverage could be devised, but this would necessitate more trial-and-error testing.

9.3. Three-Dimensional Refraction Experiments

Applying different algorithms to a nonlinear inverse problem has the advantage of providing greater confidence in those features common to the final models. By applying both methods to a relatively dense real data set and performing a resolution test assuming there was 100% data recovery in the Faeroe Basin experiment, we have also established the potential of 3-D refraction tomography to provide well-resolved, long-wavelength velocity models for the type of experiment (in terms of shot and receiver density) that is likely to become more common in the future. We have shown that an experiment consisting of a uniform grid of OBSs and a network of shot lines positioned over the OBSs provides lateral resolution of velocity at least as good as the OBS spacing. The shot spacing (receiver spacing in land experiments) has less influence on resolution but must be sufficient to allow phase correlation, and this is dependent on the data's SNR. If the combined timing and picking errors are significantly less than for the Faeroe Basin experiment (40 ms), lateral resolution better than the OBS spacing may be achieved.

Large-scale 3-D refraction experiments have several applications: (1) when reconnaissance is conducted in a new study area since a large 3-D volume can be mapped at a fraction of the cost of a 3-D seismic reflection survey; (2) when conventional reflection methods provide poor data quality, perhaps due to near-surface, high-velocity layers or scattering; (3) when the processing of reflection data requires an accurate macrovelocity model, for example, adjacent to and beneath salt structures; and (4) when the detailed mid crustal and lower crustal velocity structure is required in tectonic studies but is poorly resolved by deep reflection data. Three-dimensional refraction tomography may also prove to be a valuable step in the processing of shallow, high-resolution seismic data in which the majority of the recorded wave field is typically in the wide-angle regime. The issue of when 3-D experiments are justified and how the results of a series of 2-D profiles compares with a 3-D survey has been addressed by *Zelt and Zelt* [1998].

In some situations the use of only first arrivals to obtain velocity structure is insufficient and a better constrained model can be achieved by considering later refractions and reflections. In this case a 3-D, multilayered model is appropriate and will best be treated using a 3-D inversion method that shares some of the characteristics of the 2-D *Zelt and Smith* [1992] approach (i.e., a multilayered parameterization, layer stripping, and simultaneous refraction and reflection inversion) but also includes a fine grid parameterization and smoothing regularization as presented in this study.

Appendix: Memory and Run Times

The application to the Faeroe Basin data consists of 53,479 travel times, 29 sources (OBSs), a forward model parameterized using 331,056 nodes, and a parameterization used in the inverse step consisting of 315,000 cells for backprojection and 35,000 cells for regularized inversion. The inverse step for each method requires the most memory, 4 Mb for backprojection and 65 Mb for regularized inversion. Table 2 presents the CPU times for each method on a Sun UltraSparc workstation.

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Figure 1. Three-dimensional Faeroe Basin experiment geometry and bathymetry. Large dots show the 29 ocean bottom seismometer (OBS) locations, and small dots along the 11 profiles show the 3800 air gun shot points.

Figure 2. Example of the Faeroe Basin refraction data plotted using reduced travel time (4.5 km/s) versus source-receiver offset. A 1-12 Hz band-pass filter has been applied. The data quality of this record section is average for the data set as a whole.

Figure 3. Complete set of 53,479 picks reduced at 4.5 km/s and plotted versus source-receiver offset without regard to position or azimuth. Solid line represents average time in 1 km offset bins.

Figure 4. (a) Preferred and alternate one-dimensional (1-D) starting velocity models. (b) Comparison of preferred starting model and 1-D average of final models obtained using regularized inversion (RI) and backprojection (BP).

Figure 5. Ray paths through (a) preferred and (b) alternate 1-D starting models. Dashed lines represent average position of water-sediment interface in the study area.

Figure 6. Preferred final models from (a) regularized inversion and (b) backprojection using the preferred starting model. Velocity field shown at y positions from 3 to 23 km as labeled. Regions not sampled by ray paths are blank. Contour interval is 0.25 km/s. Note the nonlinear scale bar.

Figure 7. Travel time residuals as a function of offset for (a) preferred starting model and (b) preferred final model from regularized inversion. For clarity, every tenth residual is plotted. All inversions of the real data presented in this paper provide a similar travel time residual plot.

Figure 8. Ray coverage in the preferred final model from (a) regularized inversion and (b) backprojection using a $1.2 \times 1.2 \times 0.4$ km cell size. For reference, OBSs and shot lines are shown over the $z = 2$ km plots. Regions not sampled by ray paths are blank. Contour values are 0, 2, 5, 10, 25, 50, 100, 250, and 500. Note the nonlinear scale bar.

Figure 9. Results of checkerboard test for (a) regularized inversion and (b) backprojection. Velocity perturbations with respect to starting model are shown for depths z from 2 to 10 km as labeled. True checkerboard pattern is ± 0.2 km/s in 6×6 km squares as indicated by the grid lines. Regions not sampled by ray paths are blank.

Figure 10. Resolvability of final model from (a) regularized inversion and (b) backprojection. Resolvability at each point equals the semblance between the exact and recovered checkerboard models (Figure 9) using a $6 \times 6 \times 1.2$ km operator. For reference, OBSs and shot lines are shown over the $z = 2$ km plots. Regions with a resolvability of less than 0.5 or not sampled by ray paths are blank. Contour interval is 0.1.

Figure 11. (a) Results of checkerboard test for regularized inversion assuming there was 100% recovery of data from all 40 OBSs deployed. Velocity perturbations with respect to starting model are shown for depths z from 2 to 10 km as labeled. True checkerboard pattern is ± 0.2 km/s in 6×6 km squares as indicated by the grid lines. (b) Corresponding resolvability of final model. Resolvability at each point equals the semblance between the exact and recovered checkerboard models using a $6 \times 6 \times 1.2$ km operator. For reference the 40 OBSs and shot lines are shown over the $z = 2$ km plot. Regions not sampled by ray paths are blank. Contour interval is 0.1.

Figure 12. Maximum difference between five final models from regularized inversion and backprojection using a range of free parameters for each method corresponding to the models shown in Plates 1, 2a, 2b, and 3a. For reference, OBSs and shot lines are shown over the $z = 2$ km plot. Regions not sampled by ray paths are blank. Contour interval is 0.1 km/s.

Plate 1. Preferred final models from (a) regularized inversion and (b) backprojection using the preferred starting model. Perturbations with respect to the starting model are shown for depths z from 2 to 10 km as labeled. For reference, OBSs and shot lines are shown over the $z = 2$ km plots. Regions not sampled by ray paths are blank. Contour interval is 0.1 km/s.

Plate 2. Final models from regularized inversion using (a) a vertical/horizontal smoothness constraint $s_z = 0.075$, (b) a vertical/horizontal smoothness constraint $s_z = 0.25$, (c) the alternate starting model and (d) synthetic data calculated from the preferred final model (Plate 1a). Regions not sampled by ray paths are blank. Contour interval is 0.1 km/s.

Plate 3. Final models from backprojection using (a) a constant horizontal/vertical moving average operator size of 4.4×1.2 km for the smoothing schedule, (b) the *Hole* [1992] backprojection method, (c) the alternate starting model and (d) synthetic data calculated from the preferred final model (Plate 1b). Regions not sampled by ray paths are blank. Contour interval is 0.1 km/s.

Plate 4. Final models from regularized inversion using a strong flatness constraint between (a) $z = 6.6$ km and $z = 8.6$ km to assess the poorly constrained depth range of the model and (b) $z = 9$ km and $z = 14$ km to assess the depth range that contains the largest velocity anomaly in the preferred final model (Plate 1). For reference OBSs and shot lines are shown over the $z = 2$ km plots. Regions not sampled by ray paths are blank. Contour interval is 0.1 km/s.

Table 1. Perturbation P and Roughness R of Final Models

Description of Run	Backprojection		Regularized Inversion	
	P, $\times 10^{-1}$	R, $\times 10^{-3}$	P, $\times 10^{-1}$	R, $\times 10^{-3}$
Variable smoothing operator[a]	0.4584	0.2254		
Constant smoothing operator	0.4248	0.2401		
Hole [1992] method	0.4872	0.3377		
$s_z = 1.000$			0.4475	0.2426
$s_z = 0.500$			0.4112	0.2151
$s_z = 0.250$			0.3929	0.2033
$s_z = 0.175$			0.3926	0.2040
$s_z = 0.125$[a]			0.3908	0.2032
$s_z = 0.100$			0.3883	0.2013
$s_z = 0.075$			0.3886	0.2017
$s_z = 0.050$			0.3945	0.2078
Alternate starting model	0.4943	0.2198	0.4380	0.2182
Inversion of synthetic data from preferred final model	0.0366[b]		0.0274[b]	
Inversion of every tenth pick	0.0276[b]		0.0190[b]	
Flatness constraint between 6.6 and 8.6 km			0.3881	0.2060
Flatness constraint between 9 and 14 km			0.3888	0.2222

The preferred starting model was used for all runs except as indicated. The perturbation and roughness are calculated using (7). All backprojection runs used a variable smoothing operator, and all regularized inversion runs use s_z=0.125 except as indicated. See text for further details.

[a] Produced the preferred final model for each method.

[b] Model perturbation was calculated with respect to the preferred final model for the particular method.

Table 2. CPU Times for Inversion of Three-Dimensional Faeroe Basin data

	Backprojection	Regularized Inversion
Forward calculation[a]	2.5	5.5
Inverse calculation[b]	1.0	8.0
Total for one iteration	3.5	13.5
Number of iterations to fit data to within noise level	9	3
Total run time	31.5	40.5

CPU times are in minutes on a Sun UltraSparc workstation.

[a] Includes times, rays, and ray and slowness sums for backprojection and times, rays, and building sparse linear system for regularized inversion.

[b] Includes model smoothing and calculation of scale factor applied to model perturbation for back-projection and solution of sparse linear system and calculation of the data misfit for four iterations to determine the free parameter for regularized inversion.